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SIGHTING DEVICE-AIDED INERTIAL NAVIGATION: FUSION WITH ADAPTATIVE KALMAN FILTERING TECHNIQUES

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Abstract. *It is well-known that stand-alone Inertial Navigation Systems (INS) are certain to have their errors diverging with time. The traditional approach for solving such inconvenience is to use Global Positioning System (GPS) as an aiding device. This paper, on the other hand, investigates the feasibility of computer vision-aided INS by means of fusion with a sighting device (SD). The traditional fusion approach with INS and an auxiliary aiding device is the Kalman filter. Under certain conditions, the Kalman filter is the optimum estimation filter under any reasonable criteria. One of these conditions is the restriction that the filter should be correctly tuned. Here, an adaptive technique is tested to tune the Kalman filter used for INS/SD fusion. Results are obtained by computer simulation. In the simulation, an UAV flies a known trajectory with inertial sensor measurements corrupted by a random constant stochastic model. The INS is periodically updated by measurement data from the sighting device. Position and velocity errors, misalignment, accelerometer bias and rate-gyro drift errors with respect to ground-truth are estimated.*

Keywords: *aided inertial navigation, data fusion, Kalman filter*

1. INTRODUCTION

It is well-known (Kayton and Fried, 1997) that stand-alone Inertial Navigation Systems (INS) are certain to have their errors diverging with time. This property sets an upper bound on the duration of autonomous operations and thus such system becomes improper for use in low-cost Unmanned Aerial Vehicle (UAV) missions. The traditional approach for solving such inconvenience is to use Global Positioning System (GPS) as an aiding device (Farrell and Barth, 1999). Hence, INS/GPS fusion yields bounded attitude and navigation errors. This paper, on the other hand, investigates the feasibility of computer vision-aided INS by means of fusion with a sighting device (SD) for use in outdoor navigation in structured environments (DeSouza and Kak, 2002). In general, outdoor navigation in structured environments requires some sort of road-following (Tsugawa *et al.*, 1979; Thorpe *et al.*, 1987; Jochem *et al.*, 1995). Here, landmarks with known positions are detected by a SD so to aid the navigation system.

The traditional fusion approach with INS and an auxiliary aiding device is the Kalman filter. Under certain conditions (Maybeck, 1979), the Kalman filter is the optimum estimation filter under any reasonable criteria. One of these conditions is the restriction that the filter should be correctly tuned. Here, an adaptive technique called the Adaptative Kalman Filter (AKF) (Mohamed and Schwarz, 1999; Hide *et al.*, 2003) is tested to tune the Kalman filter used for INS/SD fusion.

The Kalman filter requires a SD model that relates imaging measurements with the dynamic model of INS and inertial sensor errors. The INS and inertial sensors errors are modeled. The sighting device is modeled through the following strategy (Bar-Itzhack, 1978): the camera motorized gimbals try to make the line of sight (LOS) point to targets or landmarks with known position. As a consequence of the navigation errors, the target/landmark is not going to be positioned at the image center. Instead, it is going to be displaced at a certain distance apart from the center. This distance is related to the navigation and inertial sensors errors and will be used as an observation equation for the Kalman filter. Other navigation algorithms (Betke and Gurvits, 1997) require as well to measure the landmarks bearings relative to each other. This is not the case in this work.

In possession of the inertial sensors error models, the extended Kalman filter is formulated to estimate the navigation and inertial sensors errors, calibrate the inertial sensors in flight, and correct the estimated attitude, velocity, and position output by the stand-alone INS.

Results are obtained by computer simulation. An UAV flies a known trajectory with inertial sensor measurements corrupted by a random constant stochastic model. The INS is periodically updated by measurement data from the sighting device. Position and velocity errors, misalignment, accelerometer bias and rate-gyro drift errors with respect to ground-truth are estimated. The performance of the adaptative filter is then compared with the off-line tuned Kalman Filter.

The approach in this work does not need the vehicle to maneuver or to comply with hard trajectory constraints. As long as the vehicle's on-board camera can see the landmark, there is no need to maneuver, in sharp contrast with INS/GPS fusion strategy (Goshen-Meskin and Bar-Itzhack, 1992; Bar-Itzhack and Berman, 1988) in which alignment errors are not observable if the vehicle is not maneuvering, or the INS/SD well-known fusion, in which the vehicle must fly exactly above a known landmark.

2. MATHEMATICAL NOTATION

The following notation (Tenenbaum, 2006) is used when describing Kinematic Motion in this document: ${}^{R}\vec{x}^{P/Q}$, where $\vec{x}$ denotes the desired property ($\vec{p}$ for position, $\vec{v}$ for velocity, $\vec{a}$ for acceleration, $\vec{\omega}$ for angular velocity, $\vec{\alpha}$ for angular acceleration), R denotes the reference frame and P denotes the particle, point, body or reference frame for which the property is referred to in relation to particle or point Q. For example, ${}^{T}\vec{v}^{P/Q}$ denotes the velocity of point P in relation to point Q taken in reference frame T, whereas ${}^{A}\vec{\omega}^{B}$ denotes angular velocity of frame B in respect to frame A.

The preceding notation does not make any reference to which coordinate frame the vector is described in. If it is desired to describe its coordinates in some coordinate frame $A = \{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$, then it is added a subscript and it is written

$$ {}^{T}\vec{v}_{A}^{P/Q} = \begin{pmatrix} {}^{T}v_{a1}^{P/Q} \\ {}^{T}v_{a2}^{P/Q} \\ {}^{T}v_{a3}^{P/Q} \end{pmatrix} \tag{1} $$

Moreover, differentiation of vectors depends on which reference frame they are derived. The notation used here to specify the reference frame R in which a vector $\vec{x}$ is derived in relation to some variable u is

$$ \frac{{}^{r}d}{du}\vec{x} \tag{2} $$

For example, vector $\vec{y}$ in Eq. (3) is the second order derivative in time t taken on reference frame S of the vector velocity of point P in relation to point O in reference frame E.

$$ \vec{y} \triangleq \frac{{}^{s}d^2}{dt^2}.{}^{e}\vec{v}^{p/o} \tag{3} $$

3. INS LINEAR ERROR PROPAGATION MODEL

The vehicle P on-board accelerometers measure specific acceleration. In this work, specific acceleration is defined as $\vec{A}^{p}$ in Eq. (4), where $\vec{g}^{m}(\vec{p}^{p/o})$ is the Earth gravitational field, O is the center of the Earth, which will be considered to be a fixed point in the inertial reference frame I.

$$ \vec{A}^{p} \triangleq \frac{{}^{i}d^2\vec{p}^{p/o}}{dt^2} - \vec{g}^{m} \tag{4} $$

Equation (4) can be used by the on-board computer to calculate the position of the vehicle in time given that we have $\vec{A}^{p}$. However, $\vec{A}^{p}(t)$ is corrupted by gyro drift $\vec{\epsilon}$, accelerometer bias $\vec{\nabla}$ and reference frame misalignment $\vec{\psi}(t)$ (Broxmeyer, 1964). In this scenario, the actual measurement $\vec{A}^{p'}$ from the accelerometer can be approximated by Eq. (5), for small angles of $\vec{\psi}$. This equation corresponds to a vector rotation $\vec{\psi}$ of $\vec{A}^{p}$ with the addition of bias $\vec{\nabla}$.

$$ \vec{A}^{p'} \approx \vec{\nabla} + \vec{A}^{p} - \vec{\psi} \times \vec{A}^{p} \tag{5} $$

The on-board computer calculates the trajectory of the UAV by using Eq. (4), while its measurements are corrupted as Eq. (5) shows. Merging these two equations together results in the calculated trajectory which is the solution of Eq. (6).

$$ \vec{\nabla} + \vec{A}^{p} - \vec{\psi} \times \vec{A}^{p} \triangleq \frac{{}^{i}d^2\vec{p}^{p/o}}{dt^2} - \vec{g}^{m} \tag{6} $$

Equation (7) results from subtracting Eq. (4) from Eq. (6), then changing reference frames using kinematic theorems (Tenenbaum, 2006), and approximating errors in gravity due to errors in position with Eq. (9). Equation (7) models the navigation error $\delta\vec{p} \triangleq \vec{p}^{p'} - \vec{p}^{p}$ and $\delta\vec{v} \triangleq^{e} \vec{v}^{p'} -^{e} \vec{v}^{p}$, where E is the Earth reference frame, T is the local level local north reference frame localized in the true UAV position, and $\vec{\Omega}$ Earth angular velocity in respect to inertial space.

$$ \vec{\nabla} - \vec{\psi} \times \vec{A}^{p}_{sp} = \frac{{}^{t}d^2}{dt^2}\delta\vec{p} +^{i} \vec{\alpha}^{t} \times \delta\vec{p} +^{i} \vec{\omega}^{t} \times ({}^{i}\vec{\omega}^{t} \times \delta\vec{p}) + 2{}^{i}\vec{\omega}^{t} \times \frac{{}^{t}d}{dt}\delta\vec{p} - \vec{\Omega} \times (\vec{\Omega} \times \delta\vec{p}) - \delta\vec{g} \tag{7} $$

$$\frac{{}^t d\vec{\psi}}{dt} +^i \vec{\omega}^t \times \vec{\psi} = \vec{\epsilon} \tag{8}$$

$$\delta\vec{g} \approx g_e R_e^2 \Big(- \frac{\delta\vec{p}}{||\delta\vec{p}||^3} + 3\frac{\vec{p}^P \cdot \delta\vec{p}}{||\vec{p}^P||^5}\vec{p}^P \Big) \tag{9}$$

Equation (7) is used in this work to model navigation errors and it will be used in the Kalman Filter model equations. Equation (10) is obtained when Eq. (7) is described in the C coordinate frame, which is the local level local north reference frame localized in the calculated position given by the on-board INS. D_p^b is the direction cossine matrix from coordinate frame B, fixed to UAV, to coordinate frame P, calculated coordinate frame based on gyro's measurements.

$$\dot{\vec{x}}_c = A\vec{x}_c + \vec{w}_c \tag{10}$$

where

$$\vec{x}_c = \begin{pmatrix} \delta\vec{p}_c^T & \delta\vec{v}_c^T & \psi_c^T & \vec{\nabla}_c^T & \vec{\epsilon}_c^T \end{pmatrix}^T \tag{11}$$

$$A = \begin{pmatrix} & & & 0_{3\times3} & 0_{3\times3} \\ & B_{9\times9} & & D_p^b & 0_{3\times3} \\ & & & 0_{3\times3} & -D_p^b \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{pmatrix} \tag{12}$$

$$B = \begin{pmatrix} 0 & {}^e\omega_{c3}^c & -{}^e\omega_{c2}^c & 1 & 0 & 0 & 0 & 0 & 0 \\ -{}^e\omega_{c3}^c & 0 & {}^e\omega_{c1}^c & 0 & 1 & 0 & 0 & 0 & 0 \\ {}^e\omega_{c2}^c & -{}^e\omega_{c1}^c & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -g_e/R_e & 0 & 0 & 0 & ({}^e\omega_{c3}^c + 2\Omega_{c3}) & -{}^e\omega_{c2}^c & 0 & -A_{c3}^p & A_{c2}^p \\ 0 & -g_e/R_e & 0 & -({}^e\omega_{c3}^c + 2\Omega_{c3}) & 0 & ({}^e\omega_{c1}^c + 2\Omega_{c1}) & A_{c3}^p & 0 & -A_{c1}^p \\ 0 & 0 & 2g_e/R_e & {}^e\omega_{c2}^c & -({}^e\omega_{c1}^c + 2\Omega_{c1}) & 0 & -A_{c2}^p & A_{c1}^p & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_1 & p_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -p_1 & 0 & p_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & -p_2 & -p_3 & 0 \end{pmatrix} \tag{13}$$

with

$$p_1 = ({}^e\omega_{c3}^c + \Omega_{c3}) \qquad p_2 = -{}^e\omega_{c2}^c \qquad p_3 = ({}^e\omega_{c1}^c + \Omega_{c1}) \tag{14}$$

4. SIGHTING DEVICE SENSOR MODEL

Suppose there are a number of landmarks whose positions are known. The UAV can try to point its SD to one of these landmarks using only information about the landmark's location $\vec{p}^L$ and UAV's position $\vec{p}^{p/o}$ and attitude. Because of navigation errors, the SD will not be able to maintain the landmark in the center of the image. Instead, the landmark will be displaced w (width) and h (height) away in the image plane from the center of the image. These variables are related to the navigation errors as Eq. (15) shows (Bar-Itzhack, 1978). It is assumed that $\vec{v}$ is a zero mean white noise, λ is the UAV's geographic latitude, H is its altitude, R_E is the eastbound radius of curvature of the Earth and R_N is the northbound radius of curvature of the Earth.

$$\begin{pmatrix} w \\ h \end{pmatrix} = \begin{bmatrix} H_1 | H_4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{pmatrix} \delta\vec{p}_c \\ \vec{\psi}_c \end{pmatrix} + \vec{v} \tag{15}$$

where

$$H_1 = H_2 + H_3 \qquad H_4 = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & 0 \end{bmatrix} \tag{16}$$

$$H_2 = \begin{bmatrix} d_{\beta_1}\beta_x & d_{\beta_1}\beta_y & 0 \\ d_{\alpha_2}\alpha_x & d_{\alpha_2}\alpha_y & d_{\alpha_2}\alpha_z \end{bmatrix} \qquad H_3 = \begin{bmatrix} \frac{d_{12}}{R_N+H} & -\frac{1}{R_E+H}(d_{11}+d_{13}tan\lambda) & 0 \\ \frac{d_{22}}{R_N+H} & -\frac{1}{R_E+H}d_{21} & 0 \end{bmatrix} \tag{17}$$

and

$$\begin{cases} (x_L \quad y_L \quad z_L)^T = \vec{p}^L \\ (x_0 \quad y_0 \quad z_0)^T = \vec{p}^{p/o} \\ d_{11} = cos\beta_0(z_L - z_0) \\ d_{12} = sin\beta_0(z_L - z_0) \\ d_{13} = -(cos\beta_0(x_L - x_0) + sin\beta_0(y_L - y_0)) = d\beta_1 \\ d_{21} = sin\beta_0 sin\alpha_0(z_L - z_0) - cos\alpha_0(y_L - y_0) \\ d_{22} = cos\alpha_0(x_L - x_0) - cos\beta_0 sin\alpha_0(z_L - z_0) \\ d_{\alpha_2} = cos\beta_0 cos\alpha_0(x_L - x_0) + sin\beta_0 cos\alpha_0(y_L - y_0) - sin\alpha_0(z_L - z_0) \\ d_{\beta_1} = -cos\beta_0(x_L - x_0) - sin\beta_0(y_L - y_0) \\ \alpha_x = (z_0 - z_L)(x_L - x_0)cos^2\alpha_0/d_0^3 \\ \alpha_y = (z_0 - z_L)(y_L - y_0)cos^2\alpha_0/d_0^3 \\ \alpha_z = cos^2\alpha_0/d_0 \\ \beta_x = sin(2\beta_0)/2(x_L - x_0) \\ \beta_y = -cos^2\beta_0/(x_L - x_0) \\ d_0 = \sqrt{(x_L - x_0)^2 + (y_L - y_0)^2} \\ \alpha_0 = tan^{-1}(z_L - z_0)/d_0 \\ \beta_0 = tan^{-1}(y_L - y_0)/(x_L - x_0) \end{cases} \tag{18}$$

5. KALMAN FILTER FORMULATION

The last two sections outlined equations which describe how stand-alone INS errors propagate and how SD data relates to INS errors. Equation (10) is the dynamic equation of the system and it is used by the KF to predict the state of the system, whereas Eq. (15) is the output equation and can it is used by the KF to update the state. In this first study, the KF is implemented in a feedback fashion: the filter estimatives are used to correct SD and navigation errors propagation models every $T_s = 0.01sec$ in Eq. (15) after time $t = 195sec$ in which the filter is heuristically assumed to converge.

The simulation uses the Discrete Kalman Filter (DKF) to estimate navigation errors due to the discrete-time nature of the on-board computer. The DKF has been well documented (Maybeck, 1979) and will not be outlined here for lack of space.

As for the filter tuning, this paper proposes to do it automatically. The measurement noise statistics (R_k) are considered known by sensor datasheets or experimental calibration. The estimated process noise Q_k can be obtained through the Adaptative Kalman Filter (Mohamed and Schwarz, 1999) according to Eq. (19), where N defines an estimation window, P_k^+ is the updated covariance of state matrix and ϕ is the system dynamic discrete matrix, which is obtained in this work by discretization of matrix A in Eq. (12) with sampling time $T_s = 0.01sec$. Matrix A requires entries which are updated every T_s. These entries are either obtained directly from INS sensors and the navigation algorithm itself. The update period in which the filter receives new information from the SD is $T_{SD} = 1sec$.

$$\hat{Q_k} = \frac{1}{N}\sum_{j=j_0}^{k} \Delta x_j \Delta x_j^T + P_k^+ - \phi P_{k-1}^+ \phi^T \qquad \Delta x_k = \hat{x}_k^+ - \hat{x}_k^- \tag{19}$$

6. EXPERIMENTAL RESULTS

The UAV maintains a straight by parts trajectory. The UAV executes piece-wise constant acceleration maneuvers with duration of $40sec$ each from the initial time to $600sec$. After $600sec$, the UAV maintains a straight trajectory with no

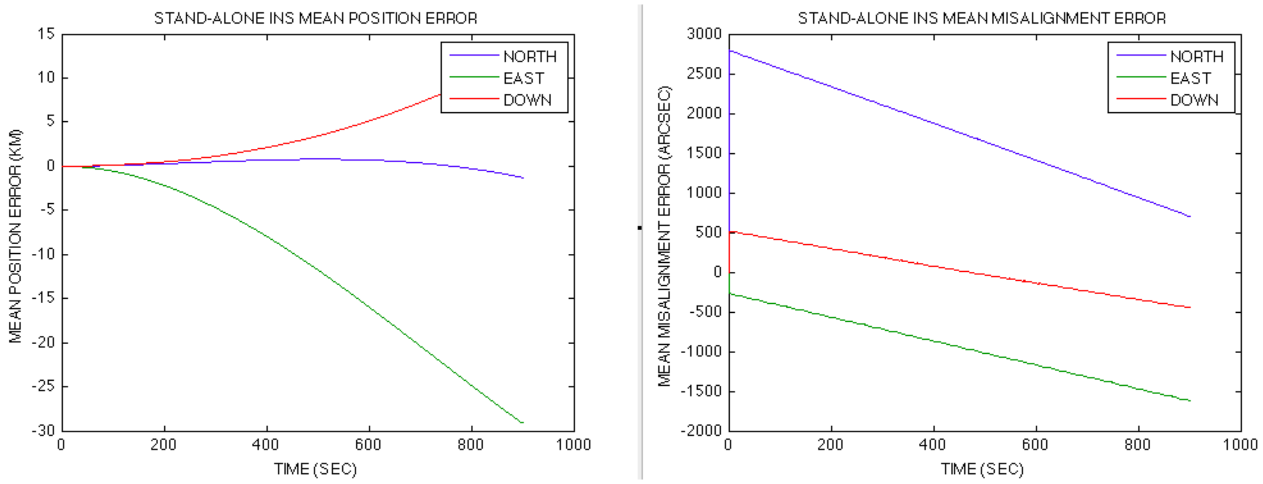

Figure 1. Mean Position and Misalignment errors for standalone INS.

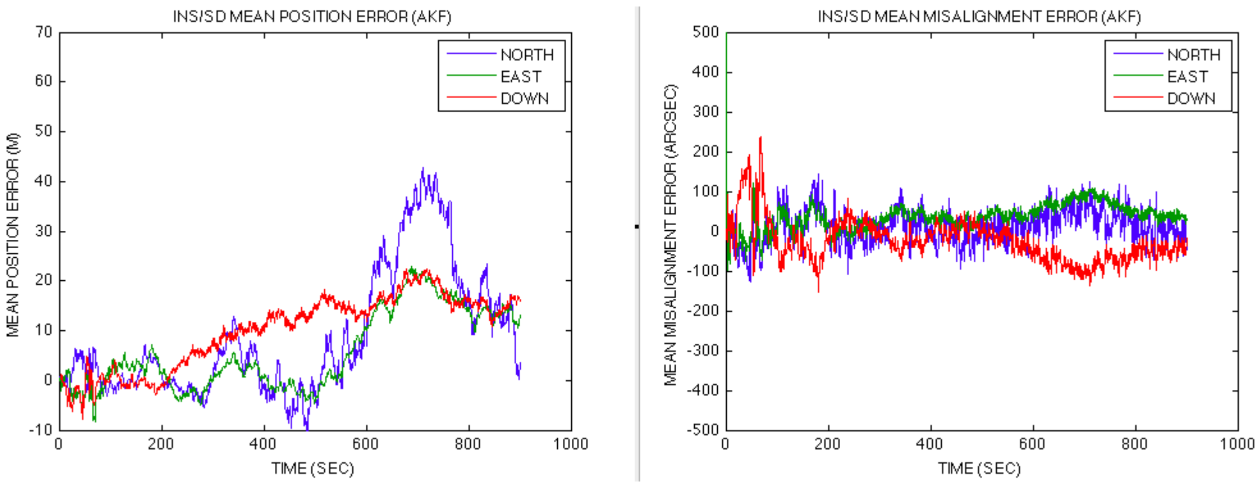

Figure 2. Mean Position and Misalignment errors for INS/SD fusion with AKF.

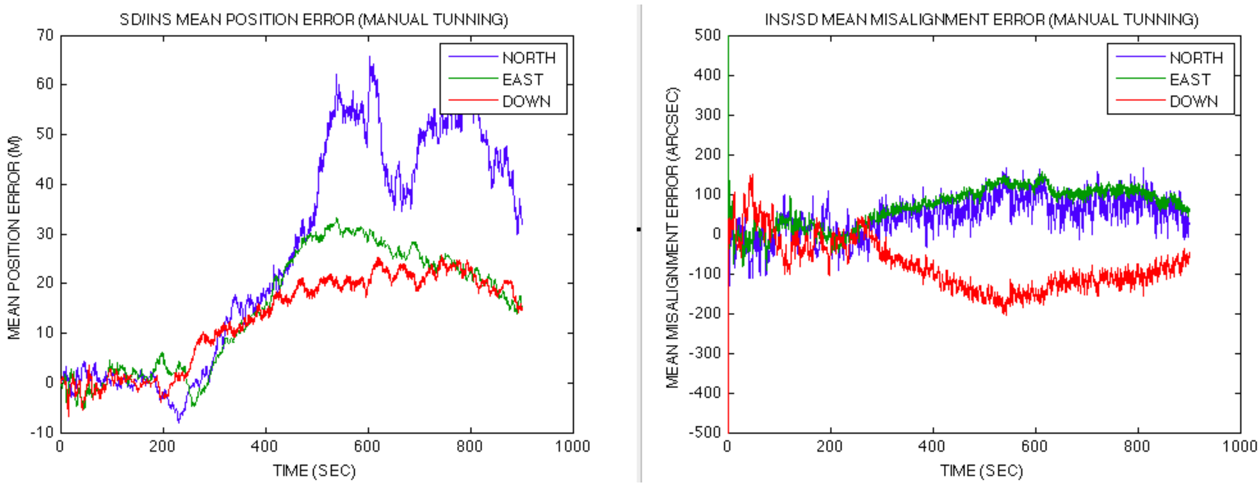

Figure 3. Mean Position and Misalignment errors for INS/SD fusion with manually tunned KF.

maneuvers until the end of the simulation ($t = 900sec$). The velocity magnitude has a linear profile that either stays constant in a value next to $30m/s$ or changes linearly with time via constant acceleration of about $0.8m/s^2$ or less.

It is assumed that the SD has always three landmarks at its disposal. In this simulation they were located at LLA coordinates (latitude, longitude and altitude) as Tab. 1 indicates. The KF has then six scalar observation equations to work with (three landmarks with SD output height and width information each).

Sensor imperfections are outlined in Tab. 1. Sensor noise variances are denoted by the symbols σ^2_{acc}, σ^2_{gyro} and σ^2_{SD}

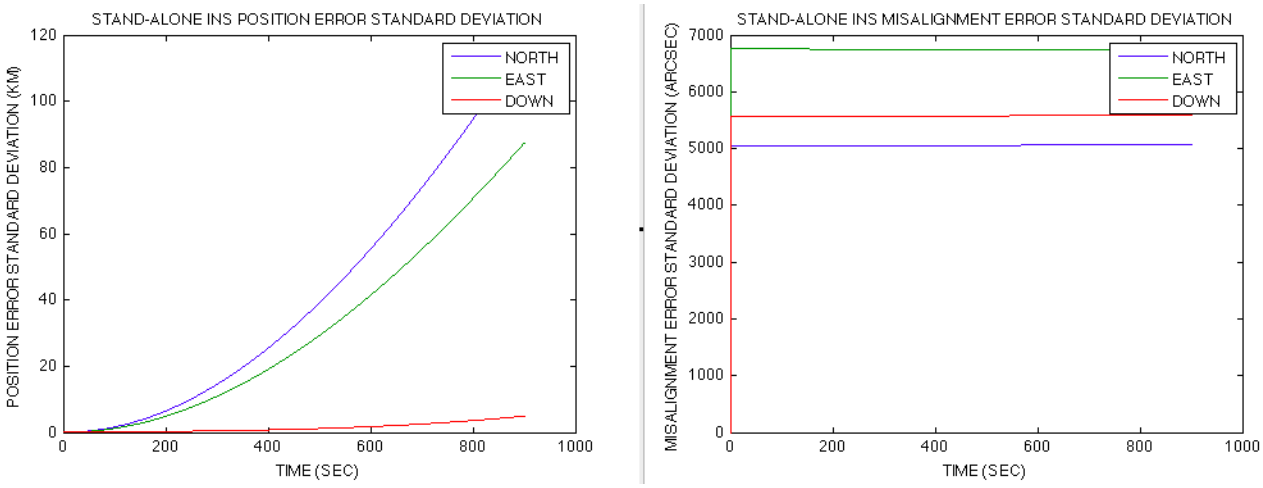

Figure 4. Position and Misalignment error standard deviation for standalone INS.

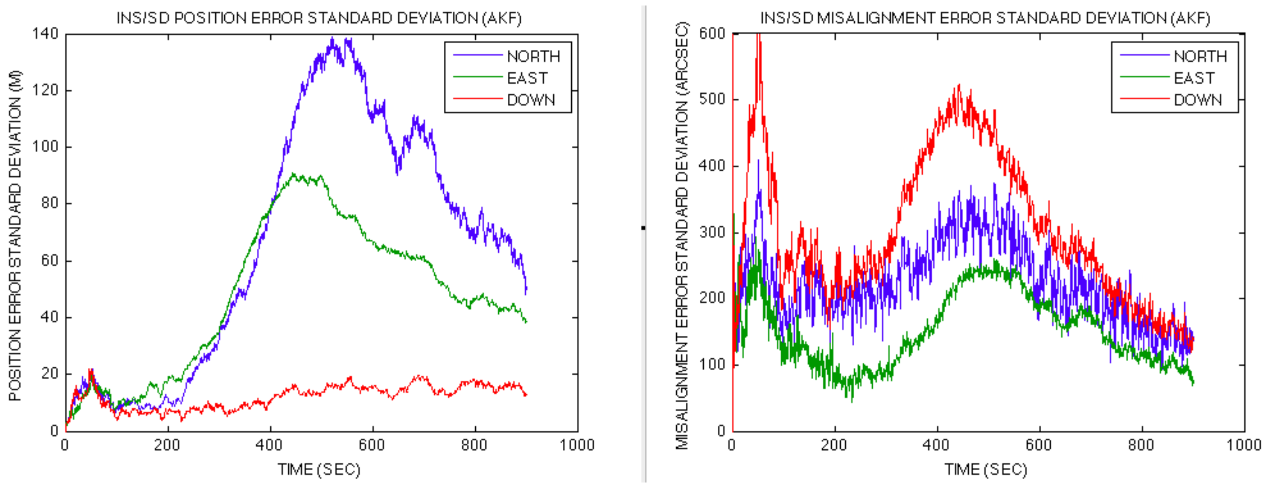

Figure 5. Position and Misalignment error standard deviation for INS/SD fusion with AKF.

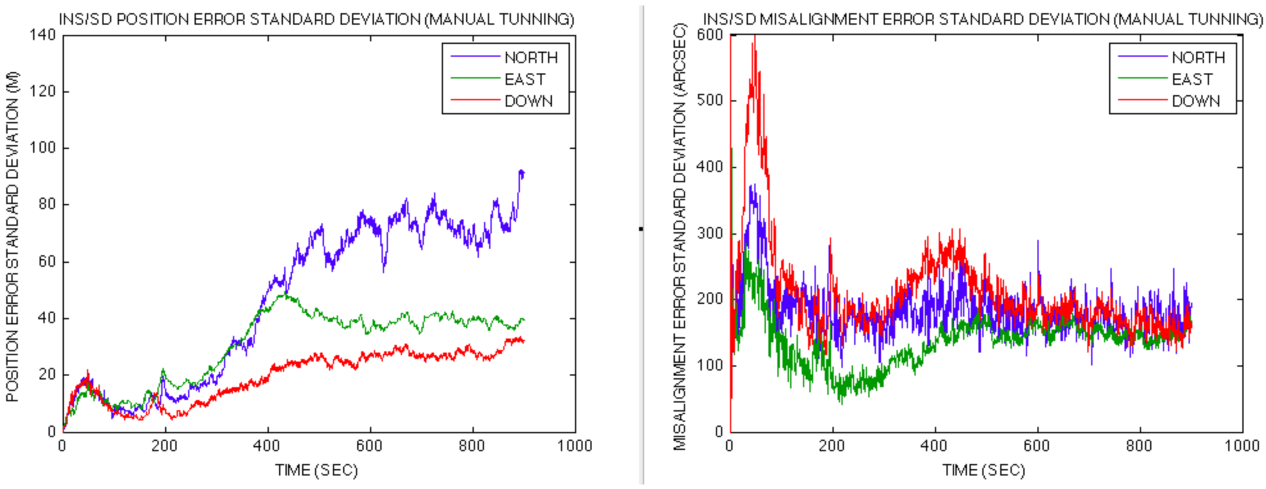

Figure 6. Position and Misalignment error standard deviation for INS/SD fusion with manually tuned KF.

for the accelerometer, gyro and SD, respectively. It is assumed that the UAV's initial position is perfectly known and its LLA coordinates are $(-23^o12', -45^o52', 6000m)$.

Monte Carlo simulations are performed. Figures 2 and 5 display the position and misalignment errors for the INS/SD AKF fusion with window size $N = 10$ and Figs. 3 and 6 display the same information with manual tuning KF. For comparison, the stand-alone INS with the exactly same sensor error statistics are plotted in Figs. 1 and 4 as well.

Table 1. Values used in simulations.

$\nabla_{ci}, i = 1..3$	$3mg$	$\epsilon_{ci}, i = 1..3$	$2^o/h$
σ^2_{acc}	$1(mg)^2$	σ^2_{gyro}	$1(^o/h)^2$
σ^2_{SD}	$40m^2$	$\vec{p}^{L1}_{LLA}$	$(-23.29^o, -45.80^o, 20m)$
$\vec{p}^{L2}_{LLA}$	$(-23.20^o, -45.85^o, 10m)$	$\vec{p}^{L3}_{LLA}$	$(-23.25^o, -45.89^o, 30m)$

7. CONCLUSIONS

Clearly, as Figs. 1 and 4 show, and it is well known, standalone INS has its errors rapidly diverging with time. Also in Figs. 1 and 4, it is also clearly indicated the vertical channel instability (Salychev, 2004), which couples with the divergence of the East channel as well. These problems motivate the use of aiding devices. In addition to the standalone INS, two different INS architectures have been tested: INS aided by SD data fusion with a conventional KF with manually tuned model covariance, and INS aided by SD data fusion with adaptive KF (AKF).

The simulation results were satisfactory when heuristically tuning the KF. The instability of the vertical channel has been fixed and the errors have become limited. Still, the errors do not converge to very small values since the sensor model H requires a rather accurate estimate of the UAV position, which has not been available just from INS/SD fusion. Thus, the incorrect sensor model leads to significative misalignment and navigation errors. However, given that the UAV after $t = 900sec$ has traveled for about $30km$, an error of about $0.1km$ gives a 0.34% relative error, which is a satisfactory navigation error for many missions.

Interestingly, the AKF had a worse performance than the heuristically tuned filter. However, given that the UAV in time $t = 900sec$ has traveled for about $30km$, an error of about $0.14km$ gives a 0.47% relative error, which is a satisfactory error for many missions. On top of that, the job of tuning the KF can be exhausting in some cases and the adaptive tool can be very helpful.

Another interesting fact observed in Fig. 2 is the limited azimuth misalignment even after time $t = 600sec$, when the UAV stops maneuvering. The use of INS/SD fusion obviates the need for maneuvers (REFS) to have all misalignment components within the observable subspace. This is one advantage over INS/GPS navigation systems. Of course, this system does not substitutes INS/GPS fusion, especially with respect to accurate position estimates. However, this work shows that INS/SD/GPS fusion can probably estimate position and misalignment without the need of maneuvers or accurate overflight over known landmarks.

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