AN IMPROVED STATIONARY FINE SELF-ALIGNMENT APPROACH FOR SINS USING MEASUREMENT AUGMENTATION

Felipe Oliveira e Silva*, Elder Moreira Hemerly*, Waldemar de Castro Leite Filho†, Ronan Arraes Jardim Chagas‡

*Instituto Tecnológico de Aeronáutica
Praça Mal. Eduardo Gomes, 50 - Vila das Acácias
São José dos Campos, SP, Brasil

†Instituto de Aeronáutica e Espaço
Praça Mal. Eduardo Gomes, 50 - Vila das Acácias
São José dos Campos, SP, Brasil

‡Instituto Nacional de Pesquisas Espaciais
Av. dos Astronautas, 1758 - Jd. Granja
São José dos Campos, SP, Brasil

Emails: felipe@ita.br, hemerly@ita.br, waldemarwclf@iae.cta.br, ronan.jardim@inpe.br

Abstract— This paper presents an alternative approach for improving the stationary fine self-alignment of strapdown inertial navigation systems (SINS). This approach is based on an expansion on the measurement vector of the linearised augmented state Kalman filter, which allows us to estimate the observable uncompensated inertial sensor biases more quickly and more accurately, contributing, thus, to increase the system performance during the navigation stage.

Keywords— Stationary fine self-alignment, Strapdown inertial navigation system, Measurement augmentation.

Resumo— Neste trabalho é apresentada uma abordagem alternativa para o auto-alinhamento fino estacionário de sistemas de navegação inercial solidários. Tal abordagem baseia-se em uma expansão do vetor de medições do Filtro de Kalman linearizado usualmente utilizado no problema do auto-alinhamento fino, o qual permite que os bias observáveis não-compensados dos sensores inerciais sejam estimados com maior precisão e rapidez, melhorando assim o desempenho global do sistema durante a fase de navegação.

Palavras-chave— Auto-alinhamento fino estacionário, Sistema de navegação inercial solidário, Expansão do vetor de medições.

1 Introduction

In general, the self-alignment procedure of a strapdown inertial navigation system consists of two steps: the coarse self-alignment and the fine self-alignment (Jekeli, 2000). The first is an analytical procedure that consists of measuring the gravity and the Earth angular velocity vectors using accelerometers and rate-gyros, respectively (Britting, 1971). Such a procedure, however, does not account for the uncompensated inertial sensor errors, specially the uncompensated biases, resulting in approximate, but incorrect, values for the estimated platform initial orientation.

The fine self-alignment, however, is a stochastic filtering and optimal estimation based procedure, implemented immediately after the coarse self-alignment, which intends to improve the estimates of the platform initial orientation, by also estimating the uncompensated sensor biases (Rogers, 2007; Grewal et al., 2013).

Among various fine self-alignment methods proposed in the literature, most of them are based on the approach originally proposed by (Bar-Itzhack and Berman, 1988). This method, summarized in Section 2, uses the own inertial navigation system (INS) and its corresponding propagation error dynamic model, linearised around a nominal operating condition, considered stationary for purposes of the self-alignment, to estimate and compensate the initial platform misalignment and the sensor biases. A linearised augmented state Kalman filter is employed.

As measurement vector, this method uses the Earth-referenced velocity error vector, which is defined as the difference between the velocities calculated by the INS, and the true velocities, assumed to be zero. As a result, one obtains a linear system with degree of observability smaller than the order of the system, reflecting the consequent inability of the filter in properly estimating all states (Wu et al., 2012).

In order to determine which are the unobservable states, different authors have studied this stationary fine self-alignment approach from the control theoretic point of view (Bar-Itzhack and Berman, 1988; Jiang and Lin, 1992), and the conclusions are not in agreement. Here, as in (Fang and Wang, 1996), it will be assumed that the unobservable states are the uncompensated north and east accelerometer biases and the east rate-gyro bias.

The objective of this paper is to propose an alternative approach for the stationary fine self-alignment of strapdown inertial navigation systems, through an expansion on the measurement vector of the linearised augmented state Kalman filter. This approach, introduced in Section 3, includes, in addition to the Earth-referenced velocity errors, the specific force and angular velocity errors, measured directly by the inertial sensors, which are directly coupled to the uncompensated sensor biases.

In Section 4, some results are presented using the traditional and the proposed stationary fine self-alignment approaches, for the simulated case of inertial sensor readings only corrupted by Gaussian white noise and uncompensated biases, with inertial platform frame perfectly aligned to the navigation frame. Finally, conclusions are presented in Section 5.

2 Traditional Fine Self-alignment

As mentioned in Section 1, one of the strategies most widely used in stationary fine self-alignment of strapdown inertial navigation systems is the one originally proposed by (Bar-Itzhack and Berman, 1988). In order to better understand this strategy, let us consider a generic INS. From (Savage, 2007), it is known that the primary function of a terrestrial inertial navigator consists in integrating, computationally, the following differential equations

$$\dot{C}_b^n = C_b^n(\boldsymbol{\omega}_{ib}^b \times) - (\boldsymbol{\omega}_{in}^n \times) C_b^n \quad (1)$$

$$\dot{\boldsymbol{v}}^n = \boldsymbol{f}^n - (\boldsymbol{\omega}_{en}^n + 2\boldsymbol{\omega}_{ie}^n) \times \boldsymbol{v}^n + \boldsymbol{g}^n \quad (2)$$

$$\dot{C}_e^n = -(\boldsymbol{\omega}_{en}^n \times) C_e^n \quad (3)$$

where $\boldsymbol{v}$ is the Earth-related platform velocity vector; $\boldsymbol{f}$ is the specific force vector measured by the accelerometers; $\boldsymbol{g}$ is the plumb-bob gravity vector; $\boldsymbol{\omega}_{ab}$ is the angular velocity vector of frame b relative to frame a, both generic frames; C_a^b is the rotation matrix (or direction cosine matrix) from frame a to frame b; the symbol $\times$ indicates the skew symmetric form of a vector; and the indexes i, e, n and b, represent the inertial frame (ECI), geographic frame (ECEF), navigation frame (UEN) and platform frame, respectively. In a more specific way, $\boldsymbol{\omega}_{en}^n$ is called transport rate vector, and $\boldsymbol{\omega}_{ib}^b$ is the angular velocity vector measured by the rate-gyros.

By using the linear perturbation technique (Rogers, 2007), one considers that, in practice, the specific force vector and the angular velocity vector measured by inertial sensors ($\tilde{}$) are corrupted by error vectors $\delta\boldsymbol{f}$ and $\delta\boldsymbol{\omega}_{ib}$, respectively, that is

$$\tilde{\boldsymbol{f}} = \boldsymbol{f} + \delta\boldsymbol{f} \quad (4)$$

$$\tilde{\boldsymbol{\omega}}_{ib} = \boldsymbol{\omega}_{ib} + \delta\boldsymbol{\omega}_{ib} \quad (5)$$

Thus, the computed vales ($\bar{}$) for the platform position, velocity and attitude become, also, equally corrupted by a term of error caused by inertial sensor errors, namely

$$\bar{C}_b^n = C_b^n + \delta C_b^n \quad (6)$$

$$\bar{\boldsymbol{v}} = \boldsymbol{v} + \delta\boldsymbol{v} \quad (7)$$

$$\bar{C}_e^n = C_e^n + \delta C_e^n \quad (8)$$

Where, by using small-angle approximation (Britting, 1971)

$$\bar{C}_b^n = [I - (\boldsymbol{\phi}\times)] C_b^n \quad (9)$$

$$\bar{C}_e^n = [I - (\boldsymbol{\theta}\times)] C_e^n \quad (10)$$

where I is the identity matrix 3×3 and $\boldsymbol{\phi}$ and $\boldsymbol{\theta}$ represent rotation vectors from computed platform attitude and position to true platform attitude and position, respectively.

Substituting (7), (9) and (10) in (1) to (3), the following set of non-linear differential equations related to the INS errors are obtained (Rogers, 2007),

$$\dot{\boldsymbol{\phi}} = \delta\boldsymbol{\omega}_{in}^n - \boldsymbol{\omega}_{in}^n \times \boldsymbol{\phi} - \delta\boldsymbol{\omega}_{ib}^n \quad (11)$$

$$\delta\dot{\boldsymbol{v}}^n = \boldsymbol{v}^n \times (\delta\boldsymbol{\omega}_{en}^n + 2\delta\boldsymbol{\omega}_{ie}^n) + \boldsymbol{f}^n \times \boldsymbol{\phi} - (\boldsymbol{\omega}_{en}^n + 2\boldsymbol{\omega}_{ie}^n) \times \delta\boldsymbol{v}^n + \delta\boldsymbol{f}^n + \delta\boldsymbol{g}^n \quad (12)$$

$$\dot{\boldsymbol{\theta}} = \delta\boldsymbol{\omega}_{en}^n - \boldsymbol{\omega}_{en}^n \times \boldsymbol{\theta} \quad (13)$$

By linearising the obtained equations around the nominal operating condition (platform considered stationary during self-alignment), and neglecting the gravity error vector $\delta\boldsymbol{g}^n$ (Jekeli, 2000), one has

$$\dot{\boldsymbol{\phi}} = \delta\boldsymbol{\omega}_{in}^n - \boldsymbol{\omega}_{in}^n \times \boldsymbol{\phi} - \delta\boldsymbol{\omega}_{ib}^n \quad (14)$$

$$\delta\dot{\boldsymbol{v}}^n = -2\boldsymbol{\omega}_{ie}^n \times \delta\boldsymbol{v}^n + \boldsymbol{f}^n \times \boldsymbol{\phi} + \delta\boldsymbol{f}^n \quad (15)$$

$$\dot{\boldsymbol{\theta}} = \delta\boldsymbol{\omega}_{en}^n \quad (16)$$

In this paper, as in (Bar-Itzhack and Berman, 1988), the self-alignment is performed when the INS is resting at a location whose geographic coordinates (latitude L) are known almost perfectly. For this reason, (16) can be eliminated from the analysis.

By representing (14) and (15) in state space form,

$$\begin{bmatrix} \delta\dot{\boldsymbol{v}}^n \\ \dot{\boldsymbol{\phi}} \end{bmatrix} = \begin{bmatrix} A_{vv} & A_{v\phi} \\ A_{\phi v} & A_{\phi\phi} \end{bmatrix} \begin{bmatrix} \delta\boldsymbol{v}^n \\ \boldsymbol{\phi} \end{bmatrix} + \begin{bmatrix} \delta\boldsymbol{f}^n \\ -\delta\boldsymbol{\omega}_{ib}^n \end{bmatrix} \quad (17)$$

where

$$A_{vv} = \begin{bmatrix} 0 & 2\Omega\cos L & 0 \\ -2\Omega\cos L & 0 & 2\Omega\sin L \\ 0 & -2\Omega\sin L & 0 \end{bmatrix} \quad (18)$$

$$A_{v\phi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -g \\ 0 & g & 0 \end{bmatrix} \quad (19)$$

$$A_{\phi v} = \begin{bmatrix} 0 & \frac{\tan L}{R} & 0 \\ 0 & 0 & -\frac{1}{R} \\ 0 & \frac{1}{R} & 0 \end{bmatrix} \quad (20)$$

$$A_{\phi\phi} = \begin{bmatrix} 0 & \Omega\cos L & 0 \\ -\Omega\cos L & 0 & \Omega\sin L \\ 0 & -\Omega\sin L & 0 \end{bmatrix} \quad (21)$$

with

$$R = \sqrt{R_L R_\lambda} \quad (22)$$

where g and Ω represent the local gravity and the Earth angular velocity, respectively, and R_L and R_λ are the meridian and transverse radii of curvature at the platform position (Farrel and Barth, 1999).

Once obtained the linearised INS error dynamic matrix, one expects to use a Kalman filter in order to estimate such errors and to provide corrections to the computed INS attitude.

As explained by (Bar-Itzhack and Berman, 1988) however, (17) is not suitable for use in Kalman filter since the accelerometer and rate-gyro errors are not white noise processes as required for proper use in a Kalman filter. Such an obstacle, however, may be overcome since the statistical characteristics of realistic accelerometer and rate-gyro errors data can be represented quite accurately by the outputs of linear models driven by white noise process.

Thus, as suggested by (Jekeli, 2000), a suitable inertial sensor error model can be seen as the sum of an uncompensated bias $\boldsymbol{b}$, which, in this paper was assumed to be a random constant, plus a Gaussian white noise $\boldsymbol{w}$, that is

$$\delta\boldsymbol{f} = \boldsymbol{b}_f + \boldsymbol{w}_f \quad (23)$$

$$\delta\boldsymbol{\omega}_{ib} = \boldsymbol{b}_\omega + \boldsymbol{w}_\omega \quad (24)$$

with

$$\dot{\boldsymbol{b}}_f = 0 \quad (25)$$

$$\dot{\boldsymbol{b}}_w = 0 \quad (26)$$

In this way, one can expand (17) in order to create an augmented INS error model that does not contain correlated noise and can be used in a Kalman filter, namely

$$\begin{bmatrix} \delta\dot{\boldsymbol{v}}^n \\ \dot{\boldsymbol{\phi}} \\ \dot{\boldsymbol{b}}_f^n \\ \dot{\boldsymbol{b}}_\omega^n \end{bmatrix} = \begin{bmatrix} A_{vv} & A_{v\phi} & I & O \\ A_{\phi v} & A_{\phi\phi} & O & -I \\ O & O & O & O \\ O & O & O & O \end{bmatrix} \begin{bmatrix} \delta\boldsymbol{v}^n \\ \boldsymbol{\phi} \\ \boldsymbol{b}_f^n \\ \boldsymbol{b}_w^n \end{bmatrix} + \begin{bmatrix} \boldsymbol{w}_f \\ \boldsymbol{w}_\omega \\ O \\ O \end{bmatrix} \quad (27)$$

or compactly,

$$\dot{\boldsymbol{X}} = A\boldsymbol{X} + \boldsymbol{W}_P \quad (28)$$

where O is a zero matrix 3×3, A is the system matrix, $\boldsymbol{X}$ is the augmented state vector and $\boldsymbol{W}_P \sim N(0, Q)$ the process noise vector.

As measurement vector $\boldsymbol{Y}$ for the Kalman filter, one uses the velocity errors, assuming that the platform is stationary (true Earth-related velocity considered to be zero). Then,

$$\boldsymbol{Y} = \begin{bmatrix} \bar{\boldsymbol{v}}^n - \boldsymbol{v}^n \end{bmatrix} \quad (29)$$

$$\boldsymbol{Y} = \begin{bmatrix} I & O & O & O \end{bmatrix} \boldsymbol{X} + \boldsymbol{W}_Y \quad (30)$$

or

$$\boldsymbol{Y} = H_Y\boldsymbol{X} + \boldsymbol{W}_Y \quad (31)$$

where H_Y is the measurement matrix and $\boldsymbol{W}_Y \sim N(0, R_Y)$ is the measurement noise vector which represents the uncertainty in the velocities assumed to be the true ones.

In order to implement the discrete Kalman filter (Brown and Hwang, 2012), the following equations are used,

$$\hat{\boldsymbol{X}}_k^- = A_d\hat{\boldsymbol{X}}_{k-1}^+ \quad (32)$$

$$P_k^- = A_d P_{k-1}^+ A_d^T + Q_d \quad (33)$$

$$\boldsymbol{K}_k = P_k^- H_Y^T (H_Y P_k^- H_Y^T + R_Y)^{-1} \quad (34)$$

$$\hat{\boldsymbol{X}}_k^+ = \hat{\boldsymbol{X}}_k^- + \boldsymbol{K}_k(\tilde{\boldsymbol{Y}}_k - H_Y\hat{\boldsymbol{X}}_k^-) \quad (35)$$

$$P_k^+ = (I_{12} - \boldsymbol{K}_k H_Y)P_k^-(I_{12} - \boldsymbol{K}_k H_Y)^T + \\ + \boldsymbol{K}_k R_Y \boldsymbol{K}_k^T \quad (36)$$

where the symbols ($\hat{}$), $(\ )^-$ and $(\ )^+$ indicate estimated, predicted and corrected values, respectively; k is the sampling time; A_d and Q_d are, respectively, the matrices A and Q discretized by Van Loan method (Farrel and Barth, 1999); P is the state covariance matrix; $\boldsymbol{K}$ is the Kalman optimal gain vector; and I_{12} is the identity matrix 12×12.

Once the states are properly estimated, one can obtain corrected estimates for the initial platform orientation and for the uncompensated biases, which are the main objectives of this algorithm, that is

$$\hat{C}_b^n = [I + (\hat{\boldsymbol{\phi}}\times)]\bar{C}_b^n \tag{37}$$

$$\hat{\boldsymbol{b}}_f^b = (\hat{C}_b^n)^{-1}\hat{\boldsymbol{b}}_f^n \tag{38}$$

$$\hat{\boldsymbol{b}}_\omega^b = (\hat{C}_b^n)^{-1}\hat{\boldsymbol{b}}_\omega^n \tag{39}$$

One of the majors performance restrictions of this approach, however, lies in the system degree of observability (Wu et al., 2012). Due to the state space expansion implemented in (27), observability matrix becomes deficient, that is

$$rank \begin{bmatrix} H_Y \\ H_Y A^2 \\ \vdots \\ H_Y A^{10} \\ H_Y A^{11} \end{bmatrix} = 9 < 12 \tag{40}$$

As mentioned earlier in Section 1, different authors have studied this stationary fine self-alignment approach from the control theoretic point of view (Bar-Itzhack and Berman, 1988; Jiang and Lin, 1992), and the conclusions are not in agreement. It can be shown, however, that for this approach, the unobservable states are the uncompensated north and east accelerometer biases b_{fy} and b_{fz}, and the east rate-gyro bias $b_{\omega y}$ (Fang and Wang, 1996).

3 Proposed Fine Self-alignment

The objective of this paper, as mentioned in Section 1, is to propose an alternative approach for the stationary fine self-alignment of strapdown inertial navigation systems. Such an approach, adapted from (Farrel and Barth, 1999), is based on an expansion on the measurement vector of the linearised augmented state Kalman filter.

In order to analyse the proposed approach, let us to consider (4), which represents the raw accelerometer readings, resolved in platform frame,

$$\tilde{\boldsymbol{f}}^b = \boldsymbol{f}^b + \delta\boldsymbol{f}^b \tag{41}$$

Naturally, in navigation frame, one has

$$\tilde{\boldsymbol{f}}^n = \boldsymbol{f}^n + \delta\boldsymbol{f}^n \tag{42}$$

where

$$\tilde{\boldsymbol{f}}^n = C_b^n\tilde{\boldsymbol{f}}^b \tag{43}$$

Analysing (43), one can conclude that the vector $\tilde{\boldsymbol{f}}^n$ can not be founded in practice, since it depends on C_b^n, whose true value is exactly what one seeks to determine. Let us to introduce a new vector $\bar{\boldsymbol{f}}^n$, which should be understood as the closest vector from $\tilde{\boldsymbol{f}}^n$ one can compute before the Kalman filtering, that is

$$\bar{\boldsymbol{f}}^n = \bar{C}_b^n\tilde{\boldsymbol{f}}^b \tag{44}$$

On the other hand, from the formal definition of the navigation frame, one can assume (as it was done to platform position and velocity) that the true value of $\boldsymbol{f}^n$ is known, and is given by (Britting, 1971)

$$\boldsymbol{f}^n = \begin{bmatrix} g \\ 0 \\ 0 \end{bmatrix} \tag{45}$$

Hence, the following difference can be computed,

$$\begin{aligned} \bar{\boldsymbol{f}}^n - \boldsymbol{f}^n &= \bar{C}_b^n\tilde{\boldsymbol{f}}^b - \boldsymbol{f}^n \\ &= [I - (\boldsymbol{\phi}\times)]C_b^n(\boldsymbol{f}^b + \delta\boldsymbol{f}^b) - \boldsymbol{f}^n \\ &= \boldsymbol{f}^n - \boldsymbol{\phi}\times\boldsymbol{f}^n + \delta\boldsymbol{f}^n - \boldsymbol{\phi}\times\delta\boldsymbol{f}^n - \boldsymbol{f}^n \end{aligned} \tag{46}$$

Neglecting error products,

$$\begin{aligned} \bar{\boldsymbol{f}}^n - \boldsymbol{f}^n &= \boldsymbol{f}^n \times \boldsymbol{\phi} + \delta\boldsymbol{f}^n \\ &= \boldsymbol{f}^n \times \boldsymbol{\phi} + \boldsymbol{b}_f^n + \boldsymbol{w}_f^n \end{aligned} \tag{47}$$

Proceeding similarly with (5), and assuming that (Britting, 1971)

$$\bar{\boldsymbol{\omega}}_{ib}^n = \bar{C}_b^n\tilde{\boldsymbol{\omega}}_{ib}^b \tag{48}$$

$$\boldsymbol{\omega}_{ib}^n = \begin{bmatrix} \Omega\sin L \\ 0 \\ \Omega\cos L \end{bmatrix} \tag{49}$$

One obtains

$$\bar{\boldsymbol{\omega}}_{ib}^n - \boldsymbol{\omega}_{ib}^n = \boldsymbol{\omega}_{ib}^n \times \boldsymbol{\phi} + \boldsymbol{b}_\omega^n + \boldsymbol{w}_\omega^n \tag{50}$$

In this way, an augmented measurement vector $\boldsymbol{Z}$ to the Kalman filter applied to stationary fine self-alignment problem is proposed, that is

$$\boldsymbol{Z} = \begin{bmatrix} \bar{\boldsymbol{v}}^n - \boldsymbol{v}^n \\ \bar{\boldsymbol{f}}^n - \boldsymbol{f}^n \\ \bar{\boldsymbol{\omega}}_{ib}^n - \boldsymbol{\omega}_{ib}^n \end{bmatrix} \tag{51}$$

$$\boldsymbol{Z} = \left[\begin{array}{c|c|c|c} I & O & O & O \\ \hline O & A_{v\phi} & I & O \\ \hline O & -A_{\phi\phi} & O & I \end{array}\right]\boldsymbol{X} + \boldsymbol{W}_Z \tag{52}$$

or

$$\boldsymbol{Z} = H_Z\boldsymbol{X} + \boldsymbol{W}_Z \tag{53}$$

where $A_{v\phi}$ and $A_{\phi\phi}$ are the matrices already defined in (19) and (21); and H_Z and $\boldsymbol{W}_Z \sim$

$N(0, R_z)$ are the new measurement matrix and new measurement noise vector, respectively.

Analysing the system degree of observability with the augmented measurement vector, one has

$$rank \begin{bmatrix} H_Z \\ H_Z A^2 \\ \vdots \\ H_Z A^{10} \\ H_Z A^{11} \end{bmatrix} = 9 < 12 \qquad (54)$$

As can be seen, the expansion of the Kalman filter measurement vector did not improve the system degree of observability. However, as will be shown in Section 4, the proposed measurement vector allows the observable states b_{fx}, $b_{\omega x}$ and $b_{\omega z}$ (up accelerometer bias, and up and north rate-gyro biases) to be more quickly and more accurately estimated than by using the traditional measurement vector.

4 Simulation Results

In order to demonstrate the superiority of the proposed stationary fine self-alignment approach upon the traditional one, some results are presented for the simulated case of inertial sensor readings only corrupted by Gaussian white noise and uncompensated biases, with inertial platform frame perfectly aligned to the navigation frame.

In the computer simulation one assumed $L = -23°12'46''$; sampling rate of $100Hz$, accelerometer readings corrupted by $0.001g$ biases and rate-gyro readings corrupted by $0.3°/h$ biases. Figures 1 to 3 illustrate the observable bias b_{fx}, $b_{\omega x}$ and $b_{\omega z}$ estimated by using each approach. As can be seen, although the proposed approach did not increase the system degree of observability, it improves the estimation convergence rate of the observable biases, which are estimated in few minutes, in contrast to few hours required by the traditional approach.

Despite the effectiveness of the proposed approach, it could not improve the convergence rate of platform misalignment vector $\boldsymbol{\phi}$, since this depends directly on the unobservable uncompensated biases b_{fy}, b_{fz} and $b_{\omega y}$, which the Kalman filter is unable to estimate. Recent works, however, have overcame this deficiency by proposing alternative procedures (Shuanbin et al., 2004), which may be combined to the approach proposed in this paper, in order to form a more robust and accurate stationary fine self-alignment algorithm for inertial navigation systems.

5 Conclusion

This paper has presented an alternative approach for improving the stationary fine self-alignment of strapdown inertial navigation systems. This approach is based on an expansion on the measurement vector of the linearised augmented state Kalman filter, which allows us to estimate the observable uncompensated inertial sensor biases more quickly and more accurately, contributing, thus, to increase INS performance during the navigation stage.

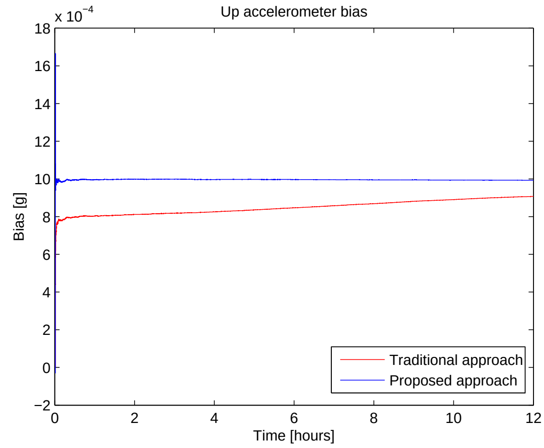

Figure 1: Estimated up accelerometer bias.

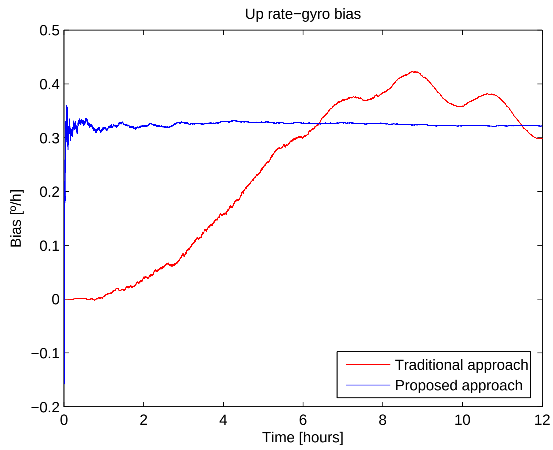

Figure 2: Estimated up rate-gyro bias.

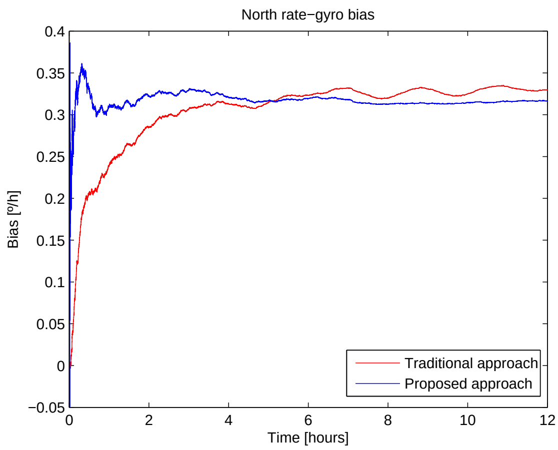

Figure 3: Estimated north rate-gyro bias.

Acknowledgement

The authors would like to thank FINEP-FUNDEP-SIA for supporting this work.

References

Bar-Itzhack, I. Y. and Berman, N. (1988). Control theoretic approach to inertial navigation systems, *Journal of Guidance and Control* **11**(3): 237–245.

Britting, K. R. (1971). *Inertial Navigation Systems Analysis*, John Wiley & Sons, Inc.

Brown, R. G. and Hwang, P. Y. C. (2012). *Introduction to Random Signals and Applied Kalman Filtering*, John Wiley & Sons, Inc.

Fang, J. C. and Wang, D. J. (1996). A fast initial alignment method for strapdown inertial navigation system on stationary base, *IEEE Transactions on Aerospace and Electronic Systems* **32**(4): 1501–1505.

Farrel, J. and Barth, M. (1999). *The Global Positioning System & Inertial Navigation*, McGraw-Hill Companies, Inc.

Grewal, M. S., Andrews, A. P. and Bartone, C. G. (2013). *Global Navigation Satellite Systems, Inertial Navigation & Integration*, John Wiley & Sons, Inc.

Jekeli, C. (2000). *Inertial Navigation Systems with Geodetic Applications*, Walter de Gruyter GmbH & Co.

Jiang, Y. F. and Lin, Y. P. (1992). Error estimation of ins ground alignment through observability analysis, *IEEE Transactions on Aerospace and Electronic Systems* **28**(1): 92–97.

Rogers, R. M. (2007). *Applied Mathematics in Integrated Navigation Systems*, American Institute of Aeronautics and Astronautics, Inc.

Savage, P. G. (2007). *Strapdown Analytics*, Strapdown Associates, Inc.

Shuanbin, Z., Weifeng, T. and Zhihua, J. (2004). A novel method improving the alignment accuracy of a strapdown inertial navigation system on a stationary base, *Measurement Science and Technology* **15**(4): 765–769.

Wu, Y., Zhang, H., Wu, M., Hu, X. and Hu, D. (2012). Observability of strapdown alignment: A global perspective, *IEEE Transactions on Aerospace and Electronics Systems* **48**(1): 78–102.